

# False vacuum inflation with a quartic potential

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We consider a variant of Hybrid Inflation, where inflation is driven by two interacting scalar fields, one of which has a ‘Mexican hat’ potential and the other a quartic potential. Given the appropriate initial conditions one of the fields can be trapped in a false vacuum state, supported by couplings to the other field. The energy of this vacuum can be used to drive inflation, which ends when the vacuum decays to one of its true minima. Depending on parameters, it is possible for inflation to proceed via two separate epochs, with the potential temporarily steepening sufficiently to suspend inflation. We use numerical simulations to analyse the possibilities, and emphasise the shortcomings of the slow-roll approximation for analysing this scenario. We also calculate the density perturbations produced, which can have a spectral index greater than one.

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## I. INTRODUCTION

Recent models of the inflationary cosmology have increasingly adopted two scalar fields, utilising the increased complexity to develop a more varied phenomenology than traditional chaotic inflation models based on a single field [1]. Where one of the fields is coupled explicitly to the Ricci scalar, one has the ‘Extended Inflation’ model [2] and its generalisations [3], which were introduced in order to revive the idea of a trapped scalar field providing the energy density to drive inflation. However, it is also perfectly possible to realise this notion without leaving Einstein gravity, by taking advantage of couplings between the two scalars [4,5]. For suitably chosen potentials, the interaction term can hold one scalar field trapped in a false vacuum state while the other evolves; eventually this evolution triggers a phase transition, which typically ends inflation within a Hubble time and leads to rapid reheating.

The simplest potential realising this ‘hybrid inflation’ scenario is<sup>1</sup>

$$V(\phi, \psi) = \frac{\lambda}{4}(\psi^2 - M^2)^2 + \frac{\lambda'}{2}\phi^2\psi^2 + \frac{1}{2}m^2\phi^2. \quad (1)$$

After its proposal by Linde [6], this potential was investigated by several authors [7–10]. The fact that it is quadratic in the inflaton field, together with the existence of the false vacuum, means that it can emerge naturally in the demanding context of superstring motivated supergravity [10]. Very few models of inflation have this distinction [11].

In this paper we consider the potential

$$V(\phi, \psi) = \frac{\lambda}{4}(\psi^2 - M^2)^2 + \frac{\lambda'}{2}\phi^2\psi^2 + \frac{\lambda''}{4}\phi^4. \quad (2)$$

It differs from the earlier one in that it is quartic instead of quadratic, in the inflaton field  $\phi$ . Though this potential does not have any clear particle physics motivation, we feel it to be worth investigating because it gives an inflation scenario with several novel features.

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<sup>1</sup>In general  $\psi$  can be multi-component with full angular symmetry, in which case its appearance in this paper should be interpreted as  $|\psi|$ .

We have not investigated the possibility that both a quartic and a quadratic term play an important role, because that would occur only in a special region of the already rather special permitted region of parameter space. A recent treatment of this more general parameter space has been carried out by Wang [12], which surveyed the possibilities for generating unusual density perturbation spectra. However, in that paper a detailed investigation of the models was not carried out; for instance the possibility of inflation proceeding in two separate epochs was not discussed. Further, as we shall show the traditional slow-roll analysis is not sufficient to outline the full range of possibilities, and some regions which possess slow-roll inflationary solutions are not accessible when one considers more general evolution. Finally in this connection we note that the particular form of potential we study is not covered by Wang's treatment as she utilised field redefinitions which are singular when the quadratic term is absent.

## II. THE TWO FIELD POTENTIAL

The absolute minima of the fields exist at  $\phi = 0$  and  $|\psi| = M$ . As the value of  $\phi^2$  rises, the minima of the  $\psi$  component at fixed  $\phi$  shift inwards until eventually, when  $\phi^2$  reaches a critical value  $\phi_c^2$ , the minima meet at  $\psi = 0$  and the symmetry is restored. The value of  $\phi_c^2$  is

$$\phi_c^2 = M^2 \frac{\lambda}{\lambda'}. \quad (3)$$

For convenience we shall always take positive  $\phi$ .

The equations of motion for the scalar fields in a flat, isotropic Friedmann universe are

$$H^2 = \frac{1}{3} \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\psi}^2 + V(\phi, \psi) \right); \quad (4)$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi, \psi)}{\partial \phi}; \quad (5)$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V(\phi, \psi)}{\partial \psi}, \quad (6)$$

where a dot indicates a derivative with respect to time. Throughout this paper units are chosen such that  $c = \hbar = m_{\text{Pl}}^2/8\pi = 1$ . These equations are invariant under a simultaneous rescaling of the potential and the time variable,  $V \rightarrow \alpha V$  and  $t \rightarrow t/\sqrt{\alpha}$ .

The initial conditions are assumed to be chaotic. We are interested primarily in the subset of initial conditions where  $\psi$  reaches its minimum at  $\psi = 0$  while  $\phi$  is still large, and then  $\phi$  evolves down the  $\psi = 0$  channel. (Initial conditions where the field falls directly into the true vacuum give phenomenology no different from single field models.) With  $\psi = 0$  and  $\phi^2 > \phi_c^2$ , the effective theory is of a single coupled field  $\phi$  in a potential

$$V(\phi) = \frac{\lambda}{4} M^4 + \frac{\lambda''}{4} \phi^4. \quad (7)$$

We shall frequently refer to the first term as the false vacuum energy. Once  $\phi^2$  reaches  $\phi_c^2$ , the  $\psi$  field develops degenerate minima which move out towards the overall minima of the system at  $\phi = 0, |\psi| = M$ . Inflation may end either when the  $\psi$  field moves away from  $\psi = 0$  (which requires  $\phi^2 < \phi_c^2$ ) or when the potential in the  $\phi$  direction becomes too steep to sustain inflation.

Since the equations of motion are invariant under a simultaneous rescaling of the potential and the time variable, it is useful to write Eq. (7) in the form

$$V = \frac{\lambda}{4} M^4 (1 + B\phi^4), \quad (8)$$

where

$$B \equiv \frac{\lambda''}{\lambda M^4}. \quad (9)$$

The rescaling guarantees that while on the  $\psi = 0$  trajectory the dynamics can depend only on  $B$ . Ignoring for the moment the instability, this potential determines the evolution of  $\phi$  through Eq. (5). One approach to its solution is given by the usual slow-roll approximation

$$3H\dot{\phi} \simeq -V', \quad (10)$$

where primes indicate derivatives with respect to  $\phi$ . Two dimensionless functions may be defined, whose values indicate the validity or otherwise of the slow-roll approximation. These functions are

$$\epsilon \equiv \frac{1}{2} \left( \frac{V'}{V} \right)^2; \quad (11)$$

$$\eta \equiv \frac{V''}{V}, \quad (12)$$

where the prime indicates a derivative with respect to  $\phi$ . A necessary, but unfortunately not sufficient, condition for the slow-roll approximation to apply is that

$$\epsilon, |\eta| \ll 1. \quad (13)$$

In any case, we shall find that the slow-roll approximation is inadequate for determining the full range of possibilities this model encompasses.

### III. INTERRUPTED INFLATION

A notable feature of the quartic potential is that inflation can end as  $\eta$  and  $\epsilon$  rise above unity, and then restart again as they fall back below unity<sup>2</sup>. Such a possibility was mentioned by Linde [8], though he was actually considering a different dynamical regime to the one we will discuss; in his scenario the field would exhibit oscillations during which the instability was ineffective, inflation restarting once the oscillations were sufficiently damped by the expansion. We shall only consider parameters where the instability acts rapidly (see Section V), implying that inflation restarts without an intermediate oscillatory regime.

For our potential, Eq. (7), we have

$$\eta = \frac{12B\phi^2}{1+B\phi^4}, \quad (14)$$

and

$$\frac{\epsilon}{\eta} = \frac{2}{3} \frac{B\phi^4}{1+B\phi^4} < \frac{2}{3}. \quad (15)$$

Since  $\eta$  is always the larger of the two quantities we shall concentrate on its value. For sufficiently large  $\phi$ ,  $\eta$  will be less than unity. As  $\phi$  falls,  $\eta$  will initially rise until it reaches a maximum beyond which it will fall back towards zero. Provided that

$$B > \frac{1}{36} \simeq 0.028, \quad (16)$$

that maximum will be above unity; the values of  $\phi$  at which  $\eta$  equals unity are then given by

$$\phi_{\pm} = \sqrt{6} \left( 1 \pm \sqrt{1 - \frac{1}{36B}} \right)^{\frac{1}{2}}. \quad (17)$$

As  $\eta$  initially rises above unity the slow-roll conditions cease to be satisfied, suggesting inflation may terminate. However, unless the instability  $\phi_c$  lies between  $\phi_+$  and  $\phi_-$ ,  $\eta$  will once more fall below unity and the slow-roll conditions shall once more be satisfied. It is therefore possible that inflation may be able to restart, though only if the slow-roll ‘attractor’ Eq. (10) can be attained in time. Unfortunately, the slow-roll approximation is necessarily a poor guide to the evolution under such circumstances [13], and we resort to numerical simulation.

Numerical simulation indicates instead that *ignoring the instability* there are three regimes of behaviour.

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<sup>2</sup>Contrary to the claim of footnote 3 in [10], such double epoch inflation can also occur for a potential of the form  $V_0 + m^2\phi^2/2$ , but there only  $\epsilon$  can fall back below unity. As a result inflation can restart only for a fraction of a Hubble time in that case.

- $0 < B \lesssim 0.19$  : Inflation proceeds as a single phase, with no interruption. Although near the top of the range both  $\eta$  and  $\epsilon$  momentarily exceed unity, they do not do so for long enough for inflation to terminate.
- $0.19 \lesssim B \lesssim 2$  : Inflation stops at  $\phi \simeq \phi_+ \simeq \sqrt{12}$ , and then restarts, giving two separate epochs. As  $B$  increases within this range, the value of  $\phi$  at which inflation restarts approaches zero.
- $2 \lesssim B$  : Inflation stops at  $\phi \simeq \phi_+ \simeq \sqrt{12}$  and never restarts.

Consequently, the regime of double epoch inflation is considerably narrower than the slow-roll approximation suggests; it needs a larger value of  $B$  and the slow-roll approximation gives no hint that there might be an upper limit on  $B$  beyond which it does not occur, especially one so close to the lower limit.

These results are important for our later classification of the possible behaviours when the presence of the instability is taken into account.

#### IV. DENSITY PERTURBATIONS

The most important constraint placed on the parameters of inflationary models comes from the size of the density perturbations that the models generate. The particular density perturbations that we are interested in are those produced about sixty  $e$ -foldings before the end of inflation, as they can induce observable microwave background anisotropies. The density contrast at this time is given by [7]

$$\delta_H^2 = \frac{1}{300\pi^2} \frac{V_{60}}{\epsilon_{60}}, \quad (18)$$

where  $V_{60}$  is the value of the potential 60  $e$ -foldings before the end of inflation and  $\epsilon_{60}$  is the value of  $\epsilon$  at the same time. Usually it is assumed that these  $e$ -foldings are  $e$ -foldings in the scale factor  $a$ . But in actual fact it is the quantity  $aH$  that must increase by 60  $e$ -folds after structure formation scales cross outside the Hubble radius. As  $H$  is approximately a constant during inflation it is usually satisfactory to consider just the increase in  $a$ , but this need not be so if inflation is interrupted.

To evaluate  $\phi_{60}$  we need the number of  $e$ -foldings occurring when inflation proceeds continuously between two values of  $\phi$

$$N(\phi_1, \phi_2) \equiv \ln \frac{a_2}{a_1} \simeq - \int_{\phi_1}^{\phi_2} \frac{V}{V'} d\phi. \quad (19)$$

If inflation proceeds without interruption after  $\phi_{60}$  then

$$N(\phi_{60}, \phi_E) = 60 = \frac{1}{8} [B^{-1}(\phi_E^{-2} - \phi_{60}^{-2}) + (\phi_{60}^2 - \phi_E^2)], \quad (20)$$

where  $\phi_E$  is the value of  $\phi$  where inflation ends. We will consider the effect of an interruption later.

The value of  $\delta_H$  as given by the recent analysis of the COBE data by Górski *et al* [14] is

$$\delta_H = 2.3 \times 10^{-5}. \quad (21)$$

This value applies as long as the spectral index  $n$  of the density perturbations is close to unity, and the amplitude of gravitational waves is sufficiently small. One is also interested in the size of the spectral index, which in the slow-roll approximation is given by

$$n = 1 + 2\eta_{60} - 6\epsilon_{60}. \quad (22)$$

#### V. THE INSTABILITY AT $\phi_C$

Many of the interesting features of this model result from the presence of the instability in the  $\psi$  field which occurs at values of  $\phi^2$  below the critical value  $\phi_c^2$ . If  $\phi^2$  is below  $\phi_c^2$ , the point  $\psi = 0$  is no longer stable and in a wide range of parameter space inflation becomes impossible. As discussed in Refs. [6,10,8] for the quadratic case, one can establish this fact by supposing that on the contrary inflation does occur, and demonstrating a contradiction. We will

do the same for the quartic potential. As with the quadratic case the regime of parameter space over which inflation is shown not to occur is large, but not necessarily optimal; in other words, it might well be possible to extend it by more detailed arguments. We will consider only the case where the false vacuum dominates the potential when the instability is encountered, since in the opposite case one does not expect the false vacuum to significantly affect the dynamics of inflation.

Suppose that inflation continues for a Hubble time after the instability is encountered. To demonstrate that this supposition leads to a contradiction we need to show that it has the following consequences

1. The  $\psi$  field rolls down to its minimum during this time.
2. Inflation does not continue in the new minimum.

As we are only trying to establish a contradiction we can suppose that the  $\phi$  and  $\psi$  fields are both homogeneous, since inhomogeneity could hardly lead to greater stability. For small values of the field  $\psi$ , the  $\psi^3$  term in Eq. (6) may be neglected, and hence the equation of motion for the  $\psi$  field is

$$\ddot{\psi} + 3H\dot{\psi} + M_\psi^2(\phi)\psi = 0, \quad (23)$$

where

$$M_\psi^2(\phi) = \lambda'(\phi^2 - \phi_c^2). \quad (24)$$

More or less independently of its initial value,  $\psi$  will roll down within a Hubble time if  $|M_\psi^2|$  exceeds  $H^2$ . Using Eq. (10) with vacuum dominated value  $H^2 = \lambda M^4/12$ , one finds that after one Hubble time

$$\phi^2 = \phi_c^2 \left( 1 + \frac{8\lambda''}{\lambda' M^2} \right), \quad (25)$$

so that

$$\frac{|M_\psi^2|}{H^2} = \frac{96\lambda''}{\lambda' M^4} \left( 1 + \frac{8\lambda''}{\lambda' M^2} \right)^{-1}. \quad (26)$$

Thus  $\psi$  will fall to its minimum within a Hubble time if

$$96 \gg \frac{\lambda' M^4}{\lambda''} + 8M^2. \quad (27)$$

In a sensible particle physics model  $M \lesssim 1$ , so this constraint is just

$$96 \gg \frac{\lambda' M^4}{\lambda''}. \quad (28)$$

It remains to show that inflation does not proceed in the new minimum. If it did,  $\phi$  would be slowly varying so the potential in the new minimum would be approximated by setting  $\phi$  equal to a constant. This gives

$$V(\phi) = \frac{\lambda''\phi^4}{4} + \frac{\lambda M^4}{4} \left( 1 - \left( 1 - \frac{\phi^2}{\phi_c^2} \right)^2 \right); \quad (29)$$

$$V'(\phi) = \lambda''\phi^3 + \lambda M^4 \frac{\phi}{\phi_c^2} \left( 1 - \frac{\phi^2}{\phi_c^2} \right); \quad (30)$$

$$V''(\phi) = 3\lambda''\phi^2 + \frac{\lambda M^4}{\phi_c^2} \left( 1 - 3\frac{\phi^2}{\phi_c^2} \right). \quad (31)$$

Inflation cannot proceed (for more than a Hubble time or so) if  $\eta \gtrsim 1$  or  $\epsilon \gtrsim 1$ , and assuming the vacuum domination condition  $\lambda''\phi_c^4 \ll \lambda M^4$  one verifies that one of these inequalities is satisfied for all  $\phi < \phi_c$  if

$$\phi_c^2 \lesssim 8. \quad (32)$$

Since the vacuum domination condition may be written  $\phi_c^4 \ll B^{-1}$ , this condition is redundant unless  $B$  is very small. Ignoring it, our conclusion is that in the vacuum dominated regime inflation cannot proceed with  $\phi^2 < \phi_c^2$ , if Eq. (28) is satisfied. We assume this condition from now on.

It should be emphasised that the nature of the phase transition that occurs when  $\phi^2$  falls below  $\phi_c^2$  is likely to be very complicated, with both the  $\psi$  and  $\phi$  fields extremely inhomogeneous [10]. There is no reason to suppose that it resembles the orderly sequence of events described above (where the slow roll of a homogeneous  $\phi$  field triggers the fast roll of a homogeneous  $\psi$  field), which was postulated only to show that a contradiction followed.

## VI. THE EPOCHS OF INFLATION

Let us now consider the possible regimes of behaviour. These are most effectively classified in accordance with the way inflation ends, with those classes subdivided according to the particular way inflation leads to that end. This description is summarised in Table I.

Let us introduce some new notation. The precise location at which inflation ceases on the  $\psi = 0$  trajectory shall be denoted  $\phi_{\text{stop}}$ . If  $B \lesssim 0.19$  it does not exist and can be allocated the default value zero. When it exists,  $\phi_+$  provides a good approximation to it. If inflation can restart further down the potential, the value at which this happens is denoted  $\phi_{\text{restart}}$ . Usually,  $\phi_-$  does *not* provide a good approximation due to the failure of the inflationary attractor. As  $B$  approaches 2, it tends to zero.

### A: Inflation ends by steepening of the potential

In this regime, inflation terminates at  $\phi_{\text{stop}}$  and does not restart. This implies  $B \gtrsim 0.19$ . The instability must be below  $\phi_{\text{stop}}$ , and if  $B \lesssim 2$ , giving the regime where inflation can potentially restart, the instability must be at  $\phi^2 > \phi_{\text{restart}}^2$ .

### B: Inflation ends by instability

1. *Single epoch inflation:* If  $B \lesssim 0.19$ ; the potential is flat enough for inflation to occur everywhere on it and instability is the only way inflation can end. If  $B \gtrsim 0.19$ , then one still has only a single epoch of inflation if the instability is positioned above  $\phi_{\text{stop}}$ .
2. *Double epoch inflation:* With  $0.19 \lesssim B \lesssim 2$ , inflation can restart provided the instability is located at small enough  $\phi$ . Phenomenologically, this is further divided by whether the scales of astrophysical interest, sixty  $e$ -foldings from the end of inflation, crossed outside the Hubble radius during the first or the second period of inflation, or both.

### A. Inflation ends by steepening of the potential

The condition that the potential can steepen enough to terminate inflation is sufficient to guarantee that the false vacuum term is small even right to the end of inflation, being at most four percent of the quartic term at  $\phi_+$ . Apart from minor corrections when  $B$  gets close to its limiting value, the situation is therefore identical to the original  $\phi^4$  model of chaotic inflation. To calculate the density perturbations we need to calculate  $\phi_{60}$ , the value of  $\phi$  sixty  $e$ -foldings before the end of inflation, which is given in this limit by

$$\phi_{60} = \sqrt{480 + \phi_+^2} = \sqrt{492}. \quad (33)$$

Hence the COBE constraint gives

$$\delta_H^2 = \frac{1}{300\pi^2} \frac{(492)^3 \lambda''}{32}, \quad (34)$$

which determines the value of  $\lambda''$  as

$$\lambda'' = 4.2 \times 10^{-13}. \quad (35)$$

This is the standard result, needing no further comment.

### B. Inflation ends by instability

#### 1. Single epoch inflation

For  $B \lesssim 0.19$  inflation can occur everywhere on the potential, and given the freedom to place the instability anywhere on it a variety of scenarios result. We shall concentrate on the two limiting cases, where the last sixty  $e$ -foldings of inflation are dominated by either the quartic term or the false vacuum term.

If  $B \gtrsim 0.19$  then we need  $\phi_c^2 > \phi_+^2 \simeq 12$ . This guarantees domination by the quartic term because during inflation  $B\phi^4 > B\phi_c^4 > 144B \gg 1$ .

With domination of the quartic term, the evolution of the system is not significantly affected by the occurrence of the instability, since the decay of the false vacuum term adds only small corrections to the dominant quartic coupling term. Hence this case is identical to the conventional chaotic inflation model and requires no further discussion.

Domination by the false vacuum term, which requires  $B \lesssim 0.19$ , is more interesting, and corresponds to the condition

$$\phi_{60}^4 \ll B^{-1}. \quad (36)$$

Under these circumstances,

$$\phi_{60} = \phi_c (1 - 480B\phi_c^2)^{-1/2}, \quad (37)$$

so false vacuum domination implies

$$480B\phi_c^2 \ll \frac{480B^{1/2}}{1 + 480B^{1/2}} < 1. \quad (38)$$

Hence  $\phi_{60} \simeq \phi_c$ . This condition may be written

$$\frac{480\lambda''}{\lambda' M^2} \ll 1, \quad (39)$$

and the vacuum domination condition becomes

$$\lambda\lambda'' \ll \lambda'^2. \quad (40)$$

The density perturbation formula gives

$$\delta_H^2 = \frac{1}{32 \times 300\pi^2} \frac{\lambda'^3 M^6}{\lambda''^2}, \quad (41)$$

which using COBE places the constraint on the parameters that

$$\lambda' M^2 = 0.037\lambda''^{2/3}. \quad (42)$$

Combining this with Eq. (39) gives

$$\lambda'' \ll 10^{-12}. \quad (43)$$

Hence this regime does not evade the usual smallness required of the quartic coupling.

In this regime  $\epsilon_{60} \ll \eta_{60}$ , so the spectral index is greater than unity. However, the vacuum domination constraint prevents its absolute value being significantly higher than unity. A more detailed investigation along the lines of Ref. [10] would be needed to examine the behaviour as one leaves perfect false vacuum domination.

## 2. Double epoch inflation

In this rather limited region of parameter space, inflation occurs in two separate epochs between which the potential is temporarily too steep to sustain inflation. Its phenomenology depends crucially on the history of the scales of astrophysical interest. There are three possibilities, any of which can be realised given suitable adjustment of  $\phi_c$ , except near the largest values of  $B$  where the second epoch may not be able to support many  $e$ -foldings between the restart and  $\phi = 0$ .

The simplest possibility is that the entire last sixty or so  $e$ -foldings occur during the second epoch of inflation; this makes the first epoch astrophysically irrelevant and is extremely similar to the single epoch inflation model discussed above, needing no further discussion here.

The most complex is where at least some of the interesting scales cross outside the Hubble radius near the end of the first epoch, *re-enter* during the interval between inflationary epochs and then exit again during the second epoch. Such a combination greatly complicates the calculation of the density perturbations, because the first Hubble radius crossing will inevitably populate some of the modes, invalidating the usual vacuum hypothesis which is used as an initial condition for the standard calculation. As a result, the spectrum produced may have an unusual amplitude, or

scale dependence, or be nongaussian. As considerable tuning is required to make the interesting scales coincide with the suspension of inflation, and because of the complexity of the scenario, we shall not address it further.

The final option is that the scales of interest crossed outside the Hubble radius for the first and only time during the first epoch of inflation.

It is important to note before progressing any further that when one talks about  $e$ -foldings, it is usually implicitly assumed that  $H$  is constant. In general, the number of  $e$ -foldings is not really a condition on the scale factor  $a(t)$ , but actually a condition on the quantity  $a(t)H(t)$ . Certainly during the suspension of inflation it is important to take this into account; a general formalism to do this is discussed in [13] though we shall not require it here.

With  $B$  in the required range, numerical simulations show that the actual reduction of  $aH$  during the intermediate regime is always small, at most around one  $e$ -folding. Consequently this aspect of the intermediate regime can be more or less ignored. [We note that for the most complex scenario mentioned above that this implies that only some of the scales of astrophysical interest can cross back inside during the intermediate era, emphasising that the results there will be far from standard.] Operationally, then, all we need do is assume that  $60 - N_1$   $e$ -foldings occur in the second epoch, where  $N_1$  would have to be determined numerically taking into account the intermediate era. Then  $\phi_{60}$  is given by

$$\phi_{60} = \sqrt{\phi_+^2 + 8N_1} = \sqrt{12 + 8N_1}. \quad (44)$$

Hence COBE gives for the value of  $\lambda''$

$$\lambda'' = \frac{5 \times 10^{-5}}{(12 + 8N_1)^3}. \quad (45)$$

Taking  $N_1 = 60$  recovers Eq. (35). In this regime the self-coupling will not be as small as in the standard single field case, though it will still be small in an absolute sense.

One must be careful though not to push this too far, because bringing  $\phi_{60}$  closer to  $\phi_+$  reduces the spectral index, which is given by

$$n = 1 - 3/N_1. \quad (46)$$

As gravitational waves are also produced, a conservative assessment of the observations [15] suggests  $n$  must exceed 0.8 and hence  $N_1$  should exceed 15.

An interesting effect of the intermediate regime is that  $H$  will be reduced during it. This offers an opportunity for  $V_{\text{end}}$  to be much lower than  $V_{60}$ ; if inflation is continuous then this cannot usually be achieved without unacceptably distorting the density perturbation spectrum. The hybrid model provides a justifiable way of inserting a feature into the potential to do this. If recent suggestions [16] that reheating may be considerably more efficient than previously thought are borne out, and given that for most models COBE implies  $V_{60}$  is in the vicinity of  $10^{16}$  GeV, such a possibility may be one way of evading the gravitino bound [17] which imposes an upper limit on the reheating temperature in supersymmetric theories<sup>3</sup>.

## VII. CONCLUSIONS

We have studied the inflationary model defined by Eq. (2) in detail. During inflation the potential is proportional to  $1 + B\phi^4$ , with the constant term arising because a non-inflaton field  $\psi$  sits in a false vacuum. When  $\phi$  falls below a critical value the vacuum is destabilised, and if it dominates inflation then ends as the fields adjust to their true vacuum values.

We have found that the delineation of the parameter space with respect to the various ways in which inflation can proceed is unexpectedly complex. In particular, there is the possibility that inflation can be interrupted and then restart, which does not occur in any model that has been studied before (discounting restart for much less than a Hubble time). What happens is that inflation ends first by the usual fast roll mechanism, and then starts again to end finally when the vacuum destabilises. We have investigated this possibility in some detail, first qualitatively using the usual slow-roll conditions, and then quantitatively by evolving the inflaton field numerically. Numerical evolution of this kind has been required very seldom in the past, but we have found it to be essential for this model.

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<sup>3</sup>Another is the false vacuum regime, where although  $V_{\text{end}}$  and  $V_{60}$  are very close, they are both very small.



An intriguing possibility, which we have noted but not explored, is that some of the cosmological scales of interest might leave the horizon during the first epoch of inflation, re-enter it and then leave it again during the second epoch. If that happened the first epoch would create inflaton particles, so that the final adiabatic density perturbation associated with the inflaton field fluctuation would be quite different from the usual form.

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TABLE I. The different dynamical regimes are indexed in accordance with the subsection in which they are discussed. Those regions marked as having no regime correspond to inaccessible parameter choices; in the first row  $\phi_{\text{stop}}$  doesn't exist and has the notional value zero, while in the last row  $\phi_{\text{restart}}$  does not exist.

	$\phi_c^2 > \phi_{\text{stop}}^2$	$\phi_{\text{stop}}^2 > \phi_c^2 > \phi_{\text{restart}}^2$	$\phi_{\text{restart}}^2 > \phi_c^2$
$B \lesssim 0.19$	<b>B1</b>	No regime	No regime
$0.19 \lesssim B \lesssim 2$	<b>B1</b>	<b>A</b>	<b>B2</b>
$2 \lesssim B$	<b>B1</b>	<b>A</b>	No regime